ARTICLE

Standards for analyzing astigmatic outcomes: Part II: Recommended statistical methods

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Surgically induced astigmatism (SIA) is the astigmatic component of the surgically induced refractive change. Vector prediction error (PE) is the key expression for evaluating refractive surgery, defined as the vector difference between the postoperatively observed refractive astigmatism and the refractive astigmatism predicted from the preoperative measurements. Astigmatic analysis of the SIA and PE includes both univariate and bivariate analyses. The magnitude of the refractive astigmatism is a predictor of visual function and a valuable component of astigmatic reporting. For vector metrics from bivariate analysis, data should be analyzed and displayed with double-angle plots. Vector calculations can be performed independent of or relative to a reference meridian or axis. Double-angle plots with convex polygons and statistical analysis for astigmatism data can be obtained easily with the wrap-up functions from the Wilcox-Holladay-Wang-Koch Statistical Software using the free software R Project for Statistical Computing or with Eyetemis software, which is also available online.

J Cataract Refract Surg 2025; 51:447–455 Copyright © 2025 Published by Wolters Kluwer on behalf of ASCRS and ESCRS

This second paper produced by the *Journal of Cataract & Refractive Surgery* astigmatism working group describes the recommended methodology for analyzing astigmatic outcomes. This approach is based on the optics of corneal and lenticular astigmatism, as well as contemporary vector analysis and statistical approaches.

The term "astigmatism" will be used throughout this section of the document to indicate *regular* astigmatism.[1–3] Surgically induced astigmatism (SIA) is the astigmatic component of the surgically induced refractive change (SIRC).[4] The SIRC can be broken down into spherical equivalent and astigmatic (SIA) components. The spherical equivalent of a refraction is characterized by a single number. By contrast, astigmatism is characterized by both a quantity and a direction.[5] As will be discussed below, astigmatic analysis of the SIA includes both univariate and bivariate analyses.

Regular astigmatism can be expressed in plus, minus, or cross-cylinder format. The plus format is used for this communication. A toric refractive surface is characterized by its orthogonal principal meridians of maximal and minimal power. The magnitude of astigmatism M ($M \geq 0$) is the absolute difference in power between these principal meridians. The direction of astigmatism Θ is defined as the meridian of most positive (or least negative) power. The astigmatism M along the meridian Θ is symbolized as (M @ Θ).

NONVECTOR METRICS

The magnitude of the refractive astigmatism—irrespective of direction—is a predictor of visual function. Therefore, comparing the magnitudes of preoperative and postoperative astigmatism or residual postoperative astigmatism from multiple independent datasets, independent of astigmatic direction, is a valuable component of astigmatic reporting. Depending on the datasets, these values should be reported:

- Mean and median refractive astigmatism with SDs and ranges.
- Percentages of eyes with postoperative refractive astigmatism (or corneal astigmatism as a substudy) within certain dioptric intervals, typically ≤0.25 diopter (D), 0.50 D, 0.75 D, 1.00 D, and >1.0 depending on the dataset. Statistical comparison of these percentages can be done using the McNemar test for dependent datasets.[6]

Submitted: October 20, 2024 | Final revision submitted: February 17, 2025 | Accepted: February 18, 2025

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Supported in part by SRB Charitable Corp., Fort Worth, Texas, the Sid W. Richardson Foundation, Fort Worth, Texas, and an unrestricted grant from Research to Prevent Blindness, New York, New York (Wang, Koch).

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0886-3350/$ - see frontmatter
https://doi.org/10.1097/j.jcrs.0000000000001645

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Although the goals of refractive corneal surgery and cataract surgery with toric intraocular lenses (IOLs) are the same (complete elimination of astigmatism), limitations of the available tools might limit our ability to achieve them. Specifically, refractive corneal surgery allows us to alter the curvature of the cornea in a continuously variable way to induce astigmatism characterized by any magnitude and direction. By contrast, most toric IOLs are available in fixed increments of astigmatic power differing by exactly 0.75 D at the IOL plane (±0.50 D at the corneal plane). The angle at which they are implanted is a continuous variable. As a result, the expected postoperative astigmatic refractive error is often not zero. In addition, the postoperative refraction is nominally measured to 0.25 D, so the tolerance would be ±0.125 D. This situation must be taken into account when predictive formulas for IOL power selection are evaluated.

Scalar analysis of astigmatic outcomes provides no information about whether a surgical procedure consistently overcorrects or undercorrects astigmatism or changes the astigmatism in a specific direction. The comparison of preoperative and postoperative magnitudes of astigmatism is the *only* relevant nonvector metric in cataract/refractive surgery. Further dissociation or separate analysis of magnitudes and direction of astigmatism is not meaningful. This is easily seen in the case of separate analysis of astigmatism direction, where the magnitude of the angular change of a 100-degree rotation of a 0.12 D cylinder cannot be compared with a 5-degree change of 8.0 D of astigmatism. Fortunately, such analyses are easily performed after conversion to vector components.

VECTOR METRICS

Single-Angle and Double-Angle Plots

Ophthalmologists and optometrists describe astigmatism with a polar coordinate system with a magnitude in diopters at a direction from 0 to 180 degrees. Described in this way, astigmatism is not a vector. Although it has magnitude, the axis or meridian has 2 directions. When we state the axis is 30 degrees, we mean the 30- to 210-degree axis (semimeridians) and use the lower value as a shorthand notation. Because an axis has 2 angles, it is not considered a Euclidean vector. A Euclidean vector must have 1 and only 1 direction.

Because astigmatism is not a vector, vector algebra may not be used to directly add obliquely crossed cylinders. Stokes recognized the problem and in 1849 made a presentation at the British association for the advancement of science in which he used graphical vector analysis with all astigmatic angles doubled.[7] In the 30- to 10-degree axis, the doubled angles would become 60 to 420 degrees and the 2 angles become the same angle (420 degrees is the same as 60 degrees). The *double-angle astigmatism vector* satisfies all our requirements for a Euclidean vector (see Figure 1 in "Standards for analyzing astigmatic outcomes: part I: astigmatism basics," this issue [page 440]).

Therefore, to perform accurate analysis, one must generate a double-angle plot in which the axis of astigmatism is doubled before the angle of astigmatism is plotted on rectangular coordinates. This is accomplished by first converting the spectacle prescription from polar (M @ Θ) to rectangular (x- and y-) coordinates. For

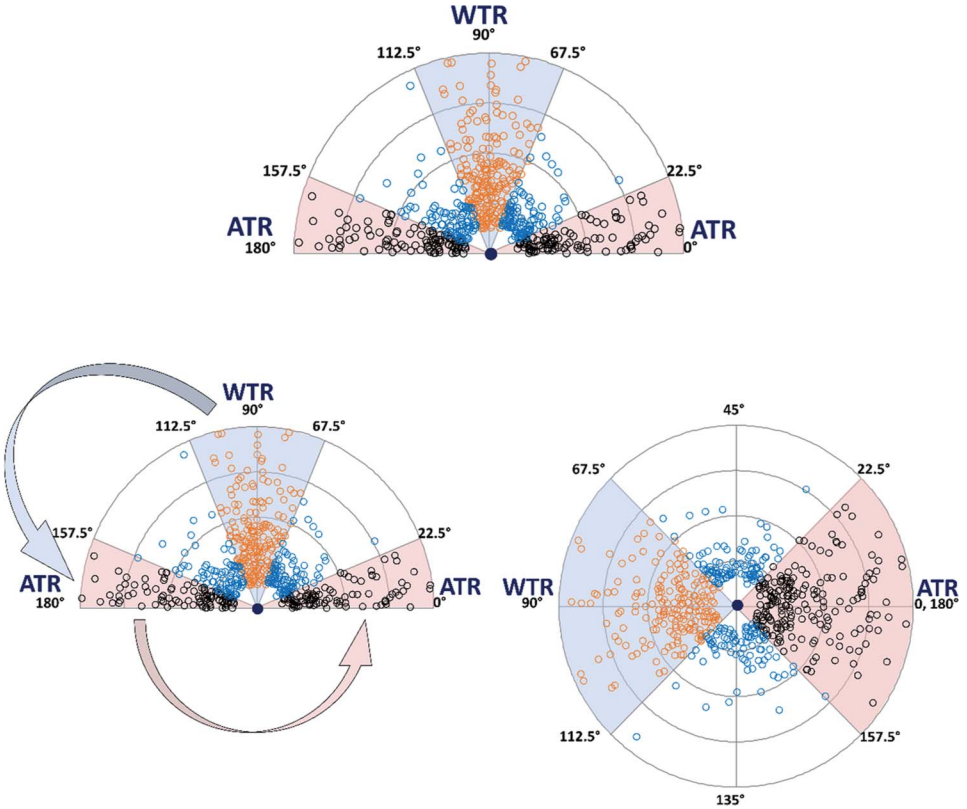

Figure 1. Single-angle plot (*top*) vs double-angle plot (*bottom*) in a sample dataset. In a double-angle plot, the WTR eyes are grouped together on the left side of the figure (*orange circles*), the oblique eyes are grouped on the top and bottom (*blue circles*), and the ATR eyes are grouped together on the right (*black circles*), allowing clear visualization and proper statistical display.[8]

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a cylinder of magnitude M and axis Θ, the 2 vector components are

$$x = M\cos 2\theta \quad (1)$$

$$y = M\sin 2\theta \quad (2)$$

The "x" value is relative to the horizontal doubled axis (x-axis), and the "y" value is relative to the vertical doubled axis (y-axis).

In Figure 1, we show the same dataset in a single-angle plot and a double-angle plot. The double-angle plot not only permits vector mathematics, as compared with single-angle plots, double-angle plots permit more intuitive interpretation of results because astigmatic trends are grouped on each side (with-the-rule to the left and against-the-rule [ATR] to the right) and not split into 2 groups along the horizontal meridian (as seen with ATR astigmatism). For these reasons, double-angle plots will be the standard for reporting astigmatic outcomes in this journal.

Vector Calculations Independent of a Reference Meridian or Axis

This method is recommended for *all* astigmatic vector analyses and is particularly useful for toric IOL outcomes, which have multiple elements contributing to SIA, making variable meridian analysis more difficult to interpret. For example, the incision might be at 180, 2 paracenteses are present, and the intended IOL meridian is 45 degrees with final position at 40 degrees. The vector prediction error (PE) that is not tied to any preoperative axis or meridian encompasses all these elements.

As indicated above, the 2 vector components are calculated using equations 1 and 2. The centroid of the data is the arithmetic mean of x and y:

$$\text{Mean of } x = \bar{x} = \frac{\sum x}{N} \quad (3)$$

$$\text{Mean of } y = \bar{y} = \frac{\sum y}{N} \quad (4)$$

In mathematics and physics, the centroid or geometric center of a plane (2-dimensional) figure ($\bar{x}$ and $\bar{y}$) is the arithmetic mean position of all the points in the figure. When the centroid is not at the origin, it represents a constant offset of the aggregate data. If an adjustment is made (as with the improvement of a toric calculator) such that the centroid is moved to the origin without changing the total variance of the data, there will be an improvement of the outcomes. In the double-angle plot figures, the $\bar{x}$ and $\bar{y}$ values for the centroid have been converted back to their polar equivalent. The resulting vector is the magnitude of the average astigmatism and its meridian or axis. The calculation of magnitude and direction (polar coordinates) of the mean astigmatism vector from x and y is

$$M_{Mean} = \bar{M} = \sqrt{\left(\frac{\sum x}{N}\right)^2 + \left(\frac{\sum y}{N}\right)^2} = \sqrt{\bar{x}^2 + \bar{y}^2} \quad (5)$$

$$\theta = \text{Arctan}\frac{\bar{M} - \frac{\sum x}{N}}{\frac{\sum y}{N}} = \text{Arctan}\frac{\bar{M} - \bar{x}}{\bar{y}} \quad (6)$$

Note that the mean value in equation 5 is the mean magnitude of the vector, which is not to be confused with the mean scalar values. For example, if we had 2 data points of 1D @ 90 degrees and 1 D @ 180 degrees, the mean magnitude of the astigmatism would be 1 D and the mean astigmatism vector would be zero because the vectors are in opposite directions.

The SDs are the square roots of the respective variances of x and y (σ_x and σ_y).

$$\text{Variance of } x = \sigma_x^2 = \frac{1}{N}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2 \quad (7)$$

$$\text{Variance of } y = \sigma_y^2 = \frac{1}{N}\sum_{i=1}^{N}\left(y_i - \bar{y}\right)^2 \quad (8)$$

$$\text{Total Variance} = \sigma_x^2 + \sigma_y^2 \quad (9)$$

$$\text{Total SD} = \sqrt{\text{Total Variance}} = \sqrt{\sigma_x^2 + \sigma_y^2} \quad (10)$$

The total variance and total SD are independent of the rotation and translation of the coordinate system for the confidence ellipses to be discussed, so they can be calculated from the original vector components. Note that total SD is not equal to the sum of the SDs of x and y (it is the square root of the sum of the squares of the SDs of x and y).

Vector Calculations Relative to a Reference Meridian or Axis

This analytical method can be used if indicated to elucidate systematic trends in the corrective algorithm that are related to the orientation of the intended surgical correction. For example, one might want to evaluate the change induced by relaxing incisions along a given meridian, or an excimer laser might systematically rotate the surgically induced astigmatic change clockwise about the preoperative astigmatic error vector. Such errors can be detected by plotting the error vectors on a graph with the horizontal axis oriented along the axis of preoperative astigmatism.

The astigmatism, (M @ Θ), is completely characterized in a new rotated coordinate system by the meridional and torsional polar values, with units in diopters: the meridional polar value x′ along the (surgical) plane Φ is the difference in meridional powers along planes Φ and (Φ + 90).[1,2]

$$\text{Meridional polar value } x' = M \times \cos(2 \times [\Theta - \Phi]) \quad (11)$$

The torsional polar value y' is the difference in meridional powers along the planes (Φ + 45) and (Φ − 45):

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$$\text{Torsional polar value } y' = M \times \sin(2 \times [\Theta - \Phi]) \quad (12)$$

The magnitude of the torsional relative to the meridional power determines the angle and direction of cylinder relative to the reference plane Φ.

The polar value system always reports the change in astigmatism along the chosen meridian. This plane is usually the surgical meridian, which again typically is the steeper corneal meridian or refractive plus axis.

1. Analysis of SIA with a single reference meridian. For instance, a surgeon consistently using a main corneal phacoemulsification incision in 90 degrees could analyze the meridional and torsional corneal powers relative to the main incision as

$$x' = M \times \cos(2 \times [\Theta - 90]) \quad (13)$$

$$y' = M \times \sin(2 \times [\Theta - 90]) \quad (14)$$

Note that this is only a partial analysis as discussed above due to the other incisions and IOL factors contributing to the SIA.

2. Analysis of SIA with a variable reference meridian. A *variable* reference plane along the preoperatively measured steep corneal meridian or refractive plus axis for each eye is typically more relevant for this approach of calculating SIA.[1–4,9–11] This direction varies for each eye, and Φ = preoperative value for Θ. All changes in astigmatism are referred to the preoperatively measured highest power of either the meridian (for cornea) or plus axis (for refraction).

In this way, one can easily ascertain the refractive result, whether perfect or undercorrected or overcorrected. For a preoperative astigmatic magnitude M and a target postoperative astigmatism of zero, the surgically induced meridional and torsional powers should be exactly –M (for elimination of M) and zero (securing no axis rotation), respectively. For the meridional power, a power reduction or an overcorrection is indicated by a negative value, while an increase in power/under correction is given by a positive value. For the torsional power, a positive value indicates a counterclockwise and a negative value clockwise rotation of the cylinder meridian.

The clinician may choose any reference meridian. The following meridians are recommended for standard surgery:

1. Corneal incisional surgery, for example, relaxing incisions: variable reference meridians along the preoperative steep corneal meridians for on-meridian corneal incisional or fixed meridians (usually 0 or 90 degrees) for standard incisions.
2. Corneal laser surgery: the axis of the preoperative refractive cylinder in plus format.

USING AND EVALUATING METRICS

The analysis is identical for corneal and refractive measurements. The clinician measures the preoperative and postoperative astigmatism and compares the latter with the predicted postoperative astigmatism. Calculations are performed with astigmatism in the double-angle format.

Vector Prediction Error

Vector PE is the key expression for evaluating surgically induced astigmatic change. It is defined as the vector difference between the postoperatively observed and the refractive astigmatism predicted from the preoperative measurements.[4] For toric IOLs, the predicted refractive astigmatism should be calculated from preoperative measurements (axial length, corneal power, and IOL power implanted) to reflect real-world outcomes and what is available to the surgeon.

PE for Refractive Astigmatism = Postoperative Refractive Astigmatism − Predicted Refractive Astigmatism

The mean magnitude of the vector PE represents the accuracy of the method that was used. Since it accounts for the centroid (trueness) and spread of the data, it should be used as the main outcome for comparing methods.[12,13] It does not necessarily correlate with uncorrected visual acuity or patient satisfaction, but it is the best measure of the performance of the methods.

Statistical Analysis of PE for Refractive Astigmatism

Statistical analysis of surgically induced changes in astigmatism is more complex than standard univariant analysis because astigmatism is bivariate and represented in a 2-dimensional space by 2 correlated values: magnitude and direction.

Normal/Nonnormal Distribution

Holladay et al. showed that astigmatic outcome data are typically not normally distributed.[14] Although one could test for normality, contemporary statistical methods are available (below) that can be used regardless of whether the data are normally distributed. Therefore, we recommend using the robust nonparametric statistics discussed below.

Absolute Values, Means, Spreads

We must now discuss the statistical principles for analysis of bivariate distributions for x and y components of the astigmatism vector. The shape of the confidence boundary is a convex polygon, as shown in Figure 2. At least 2 slightly different methodologies for analyzing non-Gaussian datasets have been proposed. These approaches produce slightly different outcomes, but both use the principles described by Holladay et al. for determining normality and using statistical methods suitable for non-Gaussian datasets.[15–17]

In the method described by Holladay et al., the analysis is a nonparametric measure of the depth of a point in a data cloud; it does not rely on the distribution being Gaussian.[16] The 95% confidence boundary for non-Gaussian distributions is a non-parametric method for finding the

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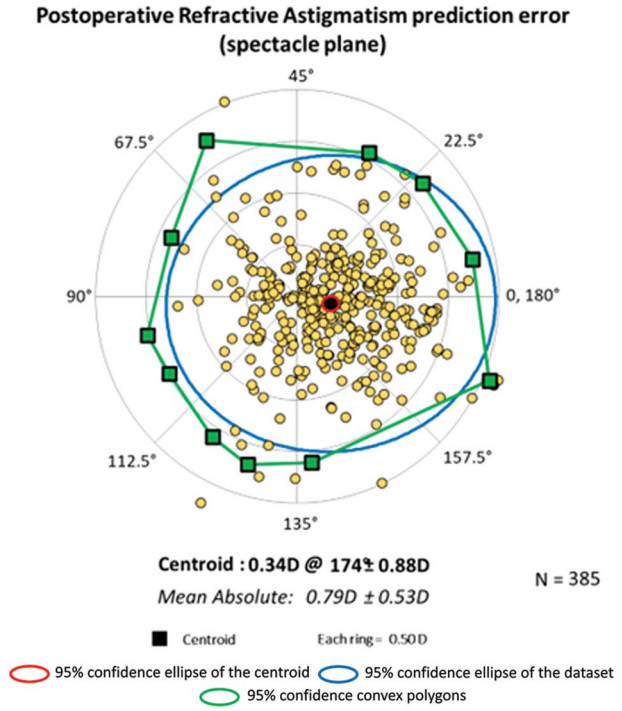

Figure 2. 95% confidence ellipses (*blue*) and 95% confidence convex polygons (*green*) calculated using a nonparametric measure of the depth of the points described above. The 95% confidence convex polygon (interior angles <180 degrees) is irregular, and the center (*reference*) is located at the *x* and *y* means.

points most tightly clustered together without making any assumptions about the type of distribution or the number of sides of the convex polygon that is needed. All points forming internal angles > 180 degrees (reflex angle) are removed leaving a convex polygon containing the 95% of the points closest to the center.

ONLINE TOOLS

Wilcox-Holladay-Wang-Koch Statistical Software With R

The double-angle plots with convex polygon described above and statistical analysis for astigmatism data can be obtained easily with the wrap-up functions from the Wilcox-Holladay-Wang-Koch (WHWK) Statistical Software using the free software R Project for Statistical Computing.[15–17] When using the wrap-up functions for the first time, files need be downloaded and installed. Download and install the R software https://www.r-project.org/, download the functions and README files from https://osf.io/xhe8u/, and install packages related to the wrap-up functions as described in detail in the 100README_-wrap_up_functions and 0000 Ophthalmology Quick Start 15dec22_V1. The dataset should be prepared as instructed in the README file. Based on the dataset, one then selects the appropriate function listed in Figure 3 and Table 1, and double-angle plots with convex polygons and statistical comparison results will be generated and saved in the same folder as the dataset file. Figures 4 and 5 and Supplemental Figures 1 and 2 (available at http://links.lww.com/JRS/B305) demonstrate the set of graphs and Excel files generated using the WHWK Statistical Software.

Eyetemis and *t* Test Analysis

The other approach is provided by Eyetemis, a software analysis tool by Abulafia and colleagues, and is available at https://www.eyetemis.com/. This tool addresses accuracy using ISO standards as being composed of 2 concepts,

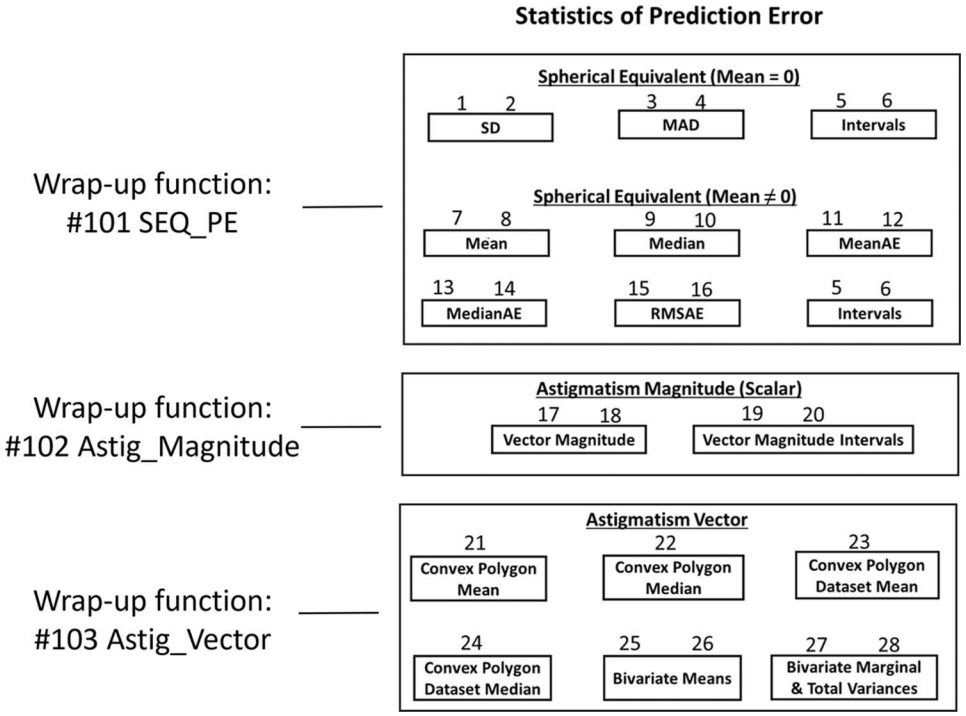

Figure 3. Flowchart of statistical analysis using the WHWK Statistical Software with R Project for Statistical Computing. WHWK = Wilcox-Holladay-Wang-Koch

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Table 1. Statistical analysis using the wrap-up functions with R Project for Statistical Computing

Parameters for analysis	Function	Outputs/statistical results
Astigmatism magnitude values • Scalar • Vector	#102 Dependent dataset: Astig_Magnitude(dependent=T) Independent dataset: Astig_Magnitude(dependent=F)	One bar graph showing percentage of eyes within intervals Two Excel files • Scalar/vector magnitude ○ Mean magnitude and number of cases ○ Nominal and adjusted *P* values for mean magnitude comparison between groups • Scalar/vector magnitude intervals ○ Number (N) and percent (%) of cases with magnitude ≤0.25 D, 0.50 D, 0.75 D, 1.00 D, 1.25 D, 1.50 D, 1.75 D, and 2.00 D ○ Nominal and adjusted *P* values for interval comparison between groups
Astigmatism vector data	#103 Dependent dataset: Astig_Vector(dependent=T) Independent dataset: Astig_Vector(dependent=F)	Two double-angle plots for each formula/method with • Mean convex polygon • Median convex polygon Six Excel files (coordinates of vertices from 4 convex polygon files used on double-angle plots) • Convex polygon mean ○ *P* value for testing hypothesis that bivariate mean of dataset is zero ○ Means for x and y, and number of cases ○ Coordinates of vertices of the convex polygon mean • Convex polygon median ○ *P* value for testing hypothesis that bivariate median of dataset is zero ○ Medians for x and y, and number of cases ○ Coordinates of vertices of the convex polygon median • Convex polygon dataset mean ○ Means for x and y, and number of groups ○ Coordinates of vertices of the convex polygon dataset mean • Convex polygon dataset median ○ Medians for x and y, and number of groups ○ Coordinates of vertices of the convex polygon dataset median • Bivariate means ○ Means for x and y ○ Nominal and adjusted *P* values for means of x and y comparison between groups • Bivariate variances ○ Variances for x and y ○ Nominal and adjusted *P* values for variance comparison between groups

trueness, and precision.[18] It uses the robust *t* test (described by Wilcox et al.) and the robust Hoteling T^2 test for univariate and bivariate analyses, respectively.[14,19] These tests include trimming of the tails to reduce the effect of outliers and Bootstrap-based calibration to account for potentially nonnormal distributions. Trueness is evaluated by comparing the centroid PE for refractive astigmatisms (PE-RAs) individually against zero and against each other by applying Robust Hoteling T^2 tests. Precision indicates the proximity between observations, and it is evaluated by comparing the trimmed-mean values of the distances between the PE-RAs and the centroid PE-RA for each method by applying the robust *t* test. Accuracy is evaluated by comparing the trimmed-mean values of the PE-RA magnitudes by applying the robust *t* test as well. The PE-RA magnitudes are also analyzed by comparing different thresholds. The graphical representation of the double-angle plots and the convex polygons are generated by the

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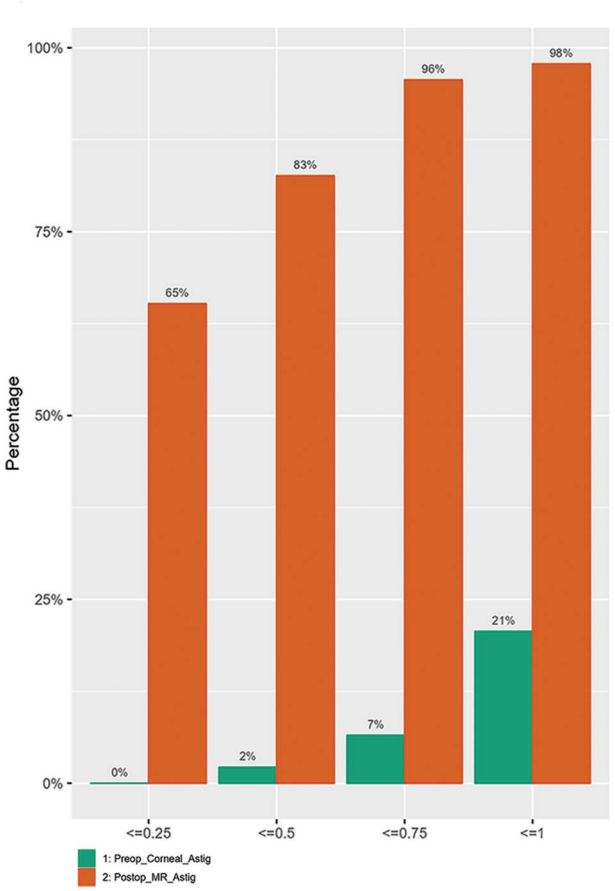

Figure 4. Graph generated using the WHWK Statistical Software with R for a sample dataset of eyes with toric IOLs. Percentage of eyes within intervals for preoperative corneal astigmatism (Preop_Corneal_astig) and postoperative manifest refractive astigmatism (Postop_MR_astig). WHWK = Wilcox-Holladay-Wang-Koch

method described by Holladay et al.[16] For estimating the power of the dataset, the software uses Monte Carlo simulations to estimate the probability of detecting a desired increase in the trimmed mean of the absolute PE-RA.

SAMPLE SIZE CALCULATION

The sample size for statistical significance will be determined by the primary outcome variable, which is typically the mean magnitude of the PE vector. For interval testing using the McNemar test, if calculating sample size for percentage differences ≥2.5% at intervals from 0.25 to 2.0 D, data from Holladay et al. indicate that approximately 250 to 400 cases are necessary to show statistical significance ($P \leq .01$).[16]

Adjusting for Multiple Comparisons on the Same Dependent Variable

P values need to be adjusted to reduce the risk for a type 1 error or false-positive result when multiple comparisons are made (centroid, total SD, intervals, or PE) using the Holm correction.[20]

CONCLUSION

Astigmatic analysis of the SIA includes both univariate and bivariate analyses. For scalar (nonvector) metrics with zero means, values that should be reported include SD, mean absolute deviation, and intervals. For scalar metrics with nonzero means, values to be reported include mean, median, mean absolute error, median AE, RMSAE, and intervals (percentages of eyes within certain dioptric intervals). For the vector metrics, data should be analyzed and displayed with double-angle plots. Vector calculations should be performed independent of a reference meridian or axis, or additionally relative to a reference meridian or axis.

For statistical analysis, the WHWK Statistical Software with R and Eyetemis analysis tool are 2 options for displaying and statistically analyzing scalar and vector outcomes of astigmatic procedures. These are the recommended methods for reporting astigmatic outcomes in JCRS.

Acknowledgments

In December 2020, the *Journal of Cataract & Refractive Surgery* (JCRS) formed a committee to reach and publish a consensus on clear and accurate astigmatism reporting guidelines to benefit JCRS authors and all researchers. The committee consisted of clinicians and researchers with expertise in this area. Following a series of virtual meetings, several of the committee members prepared the 2 manuscripts, one on background and the other on standards for analyzing astigmatism. In addition to the authors of this paper, we want to acknowledge and thank the other members of the committee who made important contributions to the concepts incorporated in the written result of this project: Graham D. Barrett, MD (Australia), Laure Gobin, PhD (Belgium), and David Smadja, MD (Israel).

WHAT WAS KNOWN

- Astigmatism analysis is complex because one must report both scalar (univariate) and vector (bivariate) outcomes.
- The vector prediction error is the primary metric for analyzing the astigmatic change produced by corneal and lenticular procedures.
- To perform vector analysis, one must generate double-angle plots in which the axes or meridians are doubled.
- Astigmatic data are often not normally distributed and require statistical methods that are appropriate for non-Gaussian datasets.

WHAT THIS PAPER ADDS

- A comprehensive approach to analyze astigmatic outcomes, including methods to report scalar and vector values, is described.
- A statistical approach that can be used regardless of whether the data are normally distributed is recommended.
- 2 astigmatism calculation tools that simplifiy the process and allow researchers to reliably calculate accurate and reproducible results are described.
- These methods are the recommended approach for astigmatic reporting in JCRS.

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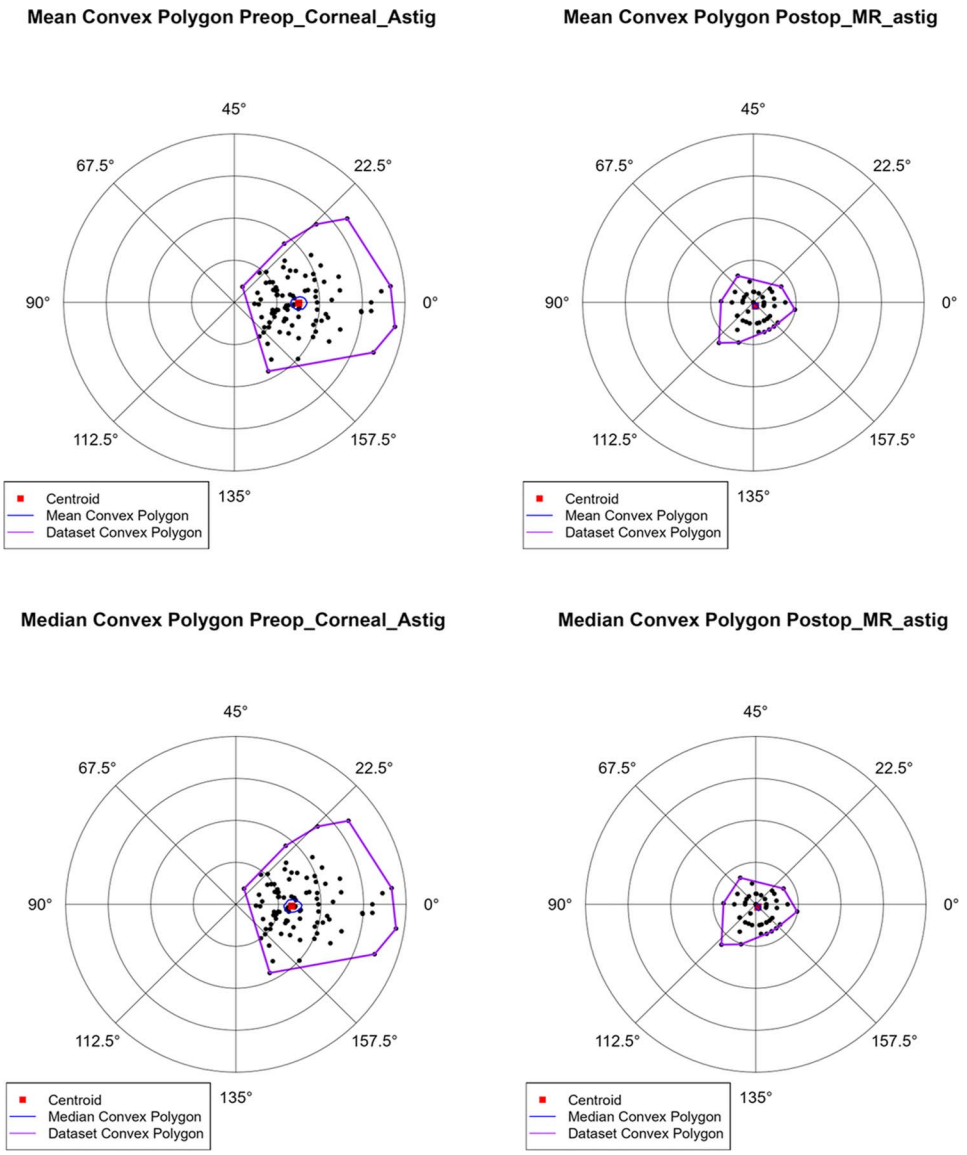

Figure 5. Graph generated using the WHWK Statistical Software with R for a sample dataset of eyes with toric IOLs. Double-angle plots with mean (*top*) and median (*bottom*) convex polygons (each ring = 1 D). WHWK = Wilcox-Holladay-Wang-Koch

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Disclosures: *T. Kohnen is a consultant, researcher and lecturer for Alcon, Oculus, Schwind, and Starr; a consultant and lecturer for Ziemer; a researcher and lecturer for Teleon Surgical; a consultant for Abbvie, Geuder, LensGen, Santen, Stadapharm, Thieme, and Carl Zeiss Meditec AG; and a lecturer for Allergan, Bausch & Lomb, Johnson & Johnson, MedUpdate, and streamedup. L. Wang is a consultant for Carl Zeiss Meditec AG and Alcon Laboratories, Inc. D.D. Koch is a consultant for Alcon Laboratories, Inc., Carl Zeiss Meditec AG, and Johnson & Johnson Vision. A. Abulafia is a consultant for Alcon Laboratories, Inc., Beyeonics Vision, Haag-Streit International, and Carl Zeiss Meditec AG. None of the other authors have any financial or proprietary interest in any material or method mentioned.*



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